MATH 231A Assignment 6

This assignment is due at 11:59 pm on Wednesday, Dec. 4, 2024.

Problem 1 (Lecture 27): If $M' \xrightarrow{i} M \xrightarrow{p} M'' \to 0$ is an exact sequence of R-modules then

$$0 \to \operatorname{Hom}_R\left(M'', N\right) \to \operatorname{Hom}_R(M, N) \to \operatorname{Hom}_R\left(M', N\right)$$

is again exact. Check this statement, in which R is any commutative ring.

Problem 2 (Exercise 27.6): Compute $H^*(\mathbb{RP}^n; R)$, where $R = \mathbb{Z}, R = \mathbb{Z}[1/2]$, and $R = \mathbb{F}_2$.

Problem 3 (Exercise 29.6): Read proof of Theorem 3.11 in Hatcher's book about the fact that the cup product renders $H^*(X)$ a commutative graded ring. You do not need to write anything for this problem.

Problem 4 (Exercise 29.7(a)): Let $n, k > 0$. Compute $H^*\left((S^k)^n; R\right)$ as an R-algebra. When k is odd, this is an "exterior algebra."

Problem 5 (Exercise 29.7(b)): $\mathbb{R}^n$ is the universal cover of $(S^1)^n = \mathbb{R}^n/\mathbb{Z}^n$. Let M be an $n \times n$ matrix with entries in $\mathbb{Z}$. It defines a linear map $\mathbb{R}^n \to \mathbb{R}^n$ in the usual way. Show that this map descends to a self-map of $(S^1)^n$. Compute the effect of this map on $H_1\left((S^1)^n\right)$ and on $H_n\left((S^1)^n\right)$.