MATH 231A Assignment 5

This assignment is due at 11:59 pm on Wednesday, Nov. 27, 2024.

Problem 1 (Exercise 20.12): Let m, n be positive integers and consider the cyclic groups $\mathbb{Z}/m$ and $\mathbb{Z}/n$. Compute the tensor product $\mathbb{Z}/m \otimes \mathbb{Z}/n$. Moreover, compute $\mathbb{Z}/m \otimes \mathbb{Z}$, $\mathbb{Z}/m \otimes \mathbb{Q}$, and $\mathbb{Z} \otimes \mathbb{Q}$.

Problem 2 (Exercise 22.5): Exercise 22.5. Let $0 \to M' \to M \to M'' \to 0$ be a short exact sequence of R-modules. Construct from it a natural long exact sequence of the form

$$\begin{tikzcd}
 & \cdots \arrow[r] & \mathrm{Tor}_2^R(M'',N) \arrow[dll] & \\
\mathrm{Tor}_1^R(M',N) \arrow[r] & \mathrm{Tor}_1^R(M,N) \arrow[r] & \mathrm{Tor}_1^R(M'',N) \arrow[dll] & \\
M' \otimes_R N \arrow[r] & M \otimes_R N \arrow[r] & M'' \otimes_R N \arrow[r] & 0.
\end{tikzcd}$$

Problem 3 (Lemma 23.11): Let $X : \mathcal{I} \to \mathbf{Ab}$ (or $\mathbf{Mod}_R$). A map $f : X \to c_L$ (given by $f_i : X_i \to L$ for $i \in \mathcal{I}$) is the direct limit if and only if:

1. For every $x \in L$, there exists an $i \in \mathcal{I}$ and an $x_i \in X_i$ such that $f_i(x_i) = x$.
2. Let $x_i \in X_i$ be such that $f_i(x_i) = 0$ in L. Then there exists some $j \geq i$ such that $f_{ij}(x_i) = 0$ in X_j.

Problem 4 (Exercise 24.3): Let X be a CW complex of finite type. Determine the dimension of $H_n(X; \mathbb{F}_p)$, for each n, in terms of the Betti numbers and torsion coefficients of $H_*(X; \mathbb{Z})$.

Problem 5: Read proofs of Theorem 3B.5 and 3B.6 in Hatcher's book. You do not need to write anything for this problem.