MATH 285Z Final Exam

This final exam is due at 11:59 pm on Wednesday, May 1, 2024. You are allowed to check lecture notes, references, and your submissions of previous assignments, but are not allowed to talk with classmates, the CA, or anyone else. You may use the theorems mentioned in the class if you state them correctly. If you want to use the results in your answers of previous submissions, please rewrite them and put them in your answer for the final exam.

Problem 1: Prove the half-lives-half-dies theorem: for any connected, oriented 3-manifold M with boundary, the map between the homology groups induced by the inclusion map

$$i_* : H_1(\partial M; \mathbb{Q}) \to H_1(M; \mathbb{Q})$$

satisfies

$$\ker(i_*) = \operatorname{im}(i_*) = \frac{1}{2} \dim H_1(\partial M; \mathbb{Q}).$$

Problem 2: From the construction of sutured Floer homology, show that

$$SFH(B^3, S^1), SHM(B^3, S^1), SHI(B^3, S^1)$$

are all 1-dimensional.

Problem 3: Let $M = S^1 \times D^2$ and let γ_n be $2n$ parallel copies of $S^1 \times$ point on the boundary, with opposite orientations for adjacent components. Compute $SFH(M, \gamma_n)$. (Hint: for large n, apply the decomposition theorem to some annuli. Note that $SFH(M_1 \sqcup M_2, \gamma_1 \sqcup \gamma_2) = SFH(M_1, \gamma_1) \otimes SFH(M_2, \gamma_2)$ for disjoint (M_1, γ_1) and (M_2, γ_2).)