MATH 285Z Assignment 6

This assignment is due at 11:59 pm on Wednesday, Apr. 24, 2024.

Problem 1: Let $M = S^3 \backslash N(K)$ be the knot complement of a knot K and let m and l be the meridian and Seifert longitude of K. Let γ_m be two copies of m with opposite orientations. Let γ_l and γ_{-m+l} be defined similarly. Show that there exists a long exact sequence obtained from the bypass exact triangle

$$\to SHI(-M, -\gamma_m) \to SHI(-M, -\gamma_l) \to SHI(-M, -\gamma_{-m+l}) \to SHI(-M, -\gamma_m) \to$$

Problem 2: Prove the following detection lemma. Suppose we have chain complexes (C_i, ∂_i) for $i \in \mathbb{Z}/3$ and chain maps $f_i : C_i \to C_{i+1}$. Suppose we have maps $h_i : C_{i-2} \to C_i$ and $k_i : C_i \to C_i$ so that

$$f_{i+1} \circ f_i = \partial_{i+2} \circ h_{i+2} + h_{i+2} \circ \partial_i$$

$$f_i \circ h_i + h_{i+1} \circ f_{i+1} = \mathrm{Id} + \partial_{i+1} \circ k_{i+1} + k_{i+1} \circ \partial_{i+1}$$

for all $i \in \mathbb{Z}/3$. Then there exists a (canonical) quasi-isomorphism from the mapping cone $\mathrm{Cone}(C_i \xrightarrow{f_i} C_{i+1})$ to C_{i+2} for all $i \in \mathbb{Z}/3$.