# MATH 285Z Assignment 2

This assignment is due at 11:59 pm on Wednesday, Mar. 6, 2024.

**Problem 1:** Let $L(p, q)$ be the lens space obtained by gluing two solid torus along their boundaries.

1. Draw a genus 1 Heegaard diagram $(\Sigma, \alpha, \beta)$ of $L(p, q)$.
2. Find basepoints $z_i, w_i$ for $i = 1, 2$ on your diagram so that $(\Sigma, \alpha, \beta, z_1, w_1)$ represents the unknot in $L(p, q)$ (i.e. the knot bounds a disk) and $(\Sigma, \alpha, \beta, z_2, w_2)$ represents the core of one of the solid torus in $L(p, q)$.
3. How many spin$^c$ structures on $L(p, q)$?

**Problem 2:** Let $K$ be the trefoil knot (either left-handed or right-handed).

1. Draw a doubly-pointed Heegaard diagram $(\Sigma, \alpha, \beta, z, w)$ for $K$ so that one of the $\beta$ curve $\beta_1$ is the meridian of $K$. Verify your construction.
2. Based on the above problem, the knot complement is obtained by attaching 2-handles along $\alpha$ and $\beta\backslash\beta_1$ on $\Sigma \times I$ and capping off the spherical boundary by a 3-ball. Draw a curve on the Heegaard surface $\Sigma$ so that it induces a longitude (a curve intersects the meridian once) on the boundary of the knot complement. It does not need to be the Seifert longitude.
3. Using the above meridian and longitude as a basis of $H_1(\partial(S^3\backslash N(K))$, describe a Heegaard diagram of the closed 3-manifold obtained from $S^3$ by the $p/q$ Dehn surgery along $K$.